Quantum response for chaotic resonances

A. Iomin
Physics Department, Technion, Haifa 32000, Israel
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Chaotic dynamics of conducting electrons in the presence of a high-frequency electromagnetic field and a constant homogeneous magnetic field is considered. It is shown that quantum fluctuations become important in this case on a time scale shorter than mean free time. Nonperturbative approach for calculation of a kinetic coefficient (conductivity) is developed. An analytical expression for the kinetic coefficient as a function of the magnetic field and a localization length is obtained. Dependence of the conductivity on the quantum localization length is studied.

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I. INTRODUCTION

Nonlinear dynamics of Bloch electrons in the presence of external fields attracts much attention today [1]. It is an example of complicated motion with possible manifestation of chaos in classical dynamics, and, consequently, realization of quantum chaos for corresponding quantum counterparts in nanostructures [2]. Essential progress in this area was achieved in the investigation of electronic motion in the lateral surface superlattices in the presence of a homogeneous magnetic field [3]. Phenomena of magnetoresistance, classical and quantum Hall conductance, and $1/f$ noise have been considered. Investigations of deviation from the classical cyclotron resonance for microwave photoconductivity in antidot arrays have been carried out as well [4].

The cyclotron resonance in metals is an example of nonlinear motion of electrons on the Fermi surface in the presence of external fields with possible manifestation of chaos as well [5]. In this case, an essential difference between chaotic and integrable dynamics leads to an essential difference in the behavior of the conductivity as a function of a magnetic field. As it has been shown in [5], conductivity oscillations as a function of a magnetic field disappear, while for the integrable case the conductivity oscillates due to resonances between cyclotronic motion and an external high-frequency field. Quantum dynamics of the chaotically interacting cyclotron resonances is considered in this paper. It is known that quantum chaotic dynamics can be totally different from classical dynamics on time scales larger than some characteristic times [6]. We study how this microscopic difference between quantum and classical dynamics reflects in difference of macroscopic characteristics such as conductivity. An important condition for observation of both isolated cyclotron resonances and interacting ones is $\omega^*\tau \gg 1$, where ω^* is the cyclotron frequency and τ is a time of a mean free path. In the case of chaotic dynamics, this condition is important for quantum-mechanical interpretation of this effect as well. Indeed, if we take into account that $\tau_H = 2\pi/\omega^*$ is a Heisenberg time over which a discrete nature of Landau spectrum becomes important and reflects in dynamics and spectroscopy [7], then $\tau \gg \tau_H$ means that the quantum effects become important on the time scale of the order of $\tau > t > \tau_H$, and some of them are a manifestation of chaotic dynamics of the classical limit. Saturation of classical diffusion in the energy space and dynamical localization of quasienergy wave functions take place due to such quantum effects [8]. Correlation functions of dynamical variables become oscillatory. One can expect that the oscillatory dependence of the surface impedance on the magnetic field is restored. It is necessary to stress that linear response theory for the perturbation discussed here does not hold in the case of the resonant interaction of the Fermi surface electrons with the perturbating microwave field in a regime where chaotic dynamics takes place in the classical limit. In this case, the Kubo formula is used for the calculation of the response of the complete chaotic system including the microwave field to a probe external field. Justification of this approach is presented here as well. Kinetic coefficients can be found in this case as a response of the chaotic system including the driving field to a small probe field. Therefore, we will consider the kinetic coefficient in the standard form of the Kubo formula

$$K(z) = \mathrm{Re}\int_0^\infty dt \mathcal{R}(t) e^{-t/\tau + izt}, \tag{1}$$

where Re is the real part and the velocity–velocity correlation function is

$$\mathcal{R}(t) = \frac{1}{2\tilde{h}} \langle \hat{v}(t)\hat{v}(0) + \hat{v}(0)\hat{v}(t) \rangle, \tag{2}$$

where $\hat{v}(t) = \hat{U}^\dagger(t)\hat{v}(0)\hat{U}(t)$ is a velocity operator in the Heisenberg representation, $\hat{U}(t)$ is an evolution operator, $\langle \cdots \rangle \equiv \mathrm{Tr}(\hat{\rho}\cdots)$ means quantum-mechanical averaging with an appropriate density operator $\hat{\rho}$, and $\tilde{h}$ is a dimensionless semiclassical parameter. In general cases, it depends both on time and temperature. This expression takes into account the driving microwave field exactly.

Therefore, in this paper we present a model in the framework of the resonant perturbation approach, which is beyond the linear response consideration. This model predicts a possible experimental realization of quantum chaos in the ''traditional'' sense, investigating quantum effects in systems that are chaotic in the classical limit ($\tilde{h} \to 0$). The following standard scenario is suggested. The main mechanism of classical chaos is an interaction between nonlinear resonances

[9,10] with the Chirikov criterion of overlap. Cyclotron resonance in metals is a nonlinear effect. In the isolated resonance approximation, the quantum dynamics of the nonlinear resonance is well described by classical equations of motion. Quantum corrections to the classical solutions are slowly growing [11] and can be omitted. The situation changes when the nonlinear resonances overlap, giving rise to chaotic dynamics. Classical equations are not valid because quantum corrections grow exponentially in time [12,7]. So-called quantum localization of classical diffusion in the energy space is a unique feature of the quantum counterpart of this chaotic system [7,8]. This quantum process is characterized by a localization length $\gamma^{-1} \propto \tilde{h}^{-2}$. We found a behavior of the kinetic coefficient K due to the localization length. When the localization length is large enough, classical chaotic dynamics dominates. In this case, the kinetic coefficient K is constant and does not reflect the magnetic-field change as a result of an overlapping of the nonlinear resonances. In the opposite extreme, when γ^{-1} is small, the dynamics reflects the discrete nature of the Landau levels. The kinetic coefficient develops poles at the quantum resonant transitions between the Landau levels and oscillates with a change of the magnetic field. Thus the appearance of these K oscillations in overlapping cyclotron resonances reflects a distinctly quantum effect. This macrocharacteristic of the system can also be measured experimentally.

The paper is organized as follows. In the Sec. II, an energy balance equation, describing an electron energy change under each passing of a skin layer, is modeled by an effective Hamiltonian for the action-angle variables. This system can be quantized by a standard semiclassical procedure. A velocity operator as well as all parameters of the effective Hamiltonian are determined in Sec. III. The quantum eigenvalue problem for quasienergies and corresponding quasienergy functions is solved in Sec. IV. It is shown that the problem corresponds effectively to one-dimensional 1D Anderson localization. A calculation of the kinetic coefficient is presented in Sec. V and Appendixes A and B. The last section contains a summary of the obtained result and a discussion of its possible experimental implementation.

II. DERIVATION OF THE EFFECTIVE HAMILTONIAN

We consider an example of the quantum effect for chaotic motion due to overlapping of cyclotron resonances in the case of the Azbel'-Kaner geometry for external fields (AKE) [13]. Hence an alternating electric field $\mathbf{E}=(0,E_0 \sin \nu t,0)$ is concentrated in a skin layer of depth Δ. Here ν is the frequency of the alternating field. Electrons move in a plane orthogonal to the homogeneous magnetic field $\mathbf{H}=(0,0,H)$ with the cyclotron frequency $\omega_c = eH/m_c c$. Here e, m_c, and c are charge, cyclotron effective mass, and the speed of light, respectively. The energy E of an electron changes by the amount $\Delta E \sim 2eE_0\sqrt{2\Delta R_c}\sin \nu t$ for every traverse of the skin layer, where R_c is the radius of the orbit. It is supposed that the time of passing of the skin layer t_Δ satisfies the following conditions: $t_\Delta \ll 2\pi/\nu \leqslant 2\pi/\omega_c$. Thus, we can construct a map connecting the energies of an electron and the phases of the electric field over the period $T=T(E)$ between two successive traverses of the skin layer. It has the following form [5,14]:

$$\begin{aligned} \tilde{\mathcal{E}}_{n+1} &= \tilde{\mathcal{E}}_n + \epsilon \sin \nu t_n, \\ t_{n+1} &= t_n + T_c(\tilde{\mathcal{E}}_{n+1}), \end{aligned} \tag{3}$$

where $\tilde{\mathcal{E}} = E/\hbar\omega^*$ is the dimensionless energy; and $\epsilon = \Delta E/\hbar\omega^*$ is a dimensionless semiclassical parameter connected to the number of energy levels captured in a resonance. In this case, the cyclotron frequency is determined by $\omega_c(\mathcal{E}) = 2\pi/\omega^* T_c(\mathcal{E})$, where ω^* is a cyclotron frequency introduced here for the dimensionless scaling and it will be determined in the explicit form in what follows. In the vicinity of a fixed point $\tilde{\mathcal{E}}^*$, which is defined by the resonance condition

$$\nu T(\tilde{\mathcal{E}}^*) = 2\pi f, \quad f = 1,2,\ldots, \tag{4}$$

Eqs. (3) can be locally approximated by a standard map [14,10]

$$\begin{aligned} \mathcal{E}_{n+1} &= \mathcal{E}_n + \epsilon \sin \nu t_n, \\ t_{n+1} &= t_n + T_0 + \zeta \mathcal{E}_{n+1}, \end{aligned} \tag{5}$$

where

$$\mathcal{E} = \tilde{\mathcal{E}} - \tilde{\mathcal{E}}^*, \quad \zeta = \frac{dT(\tilde{\mathcal{E}}^*)}{d\tilde{\mathcal{E}}^*}, \quad T_0 \equiv T(\tilde{\mathcal{E}}^*). \tag{6}$$

For long-term dynamics, all these errors accumulate. For the short time scale of the order of a period of the high-frequency perturbation $2\pi/\nu$ (upon which the quantum analysis is carried out below for the Floquet evolution operator) this round-off does not affect the dynamics, but simplifies the following analytical consideration. We notice that the main structure of the nonlinear resonances remains unchanged [15]. For the slowly varying spectrum, this procedure is the standard and widely used in the nonlinear dynamics theory [9,14–16].

The nonlinear resonances are determined by the second term in the expansion, such that the Chirikov criterion of chaos is $\mathcal{K} = \epsilon \nu \zeta \geqslant 1$.

For the semiclassical considerations, we will pass to the action-angle variables for which a procedure of the semiclassical quantization is standard [15]. In the absence of the perturbation, $\mathcal{E}$ is energy of a periodic motion with a frequency

$$\omega(I) = \frac{d\mathcal{E}}{dI} = \frac{2\pi}{T(\mathcal{E})} \approx 2\pi (T_0 + \zeta \mathcal{E})^{-1}. \tag{7}$$

Solving this equation, we obtain a solution that corresponds to the following condition $\mathcal{E} \ll T_0/\zeta$:

$$\mathcal{E} = \Omega I - \mu I^2/2, \tag{8}$$

where $\Omega = 2\pi/T_0$ and $\mu = 16\pi^2 \zeta / T_0^3$.

The energy change resulting from the perturbation over time $\Delta t = T$ reads from Eqs. (5) and (8):

$$\Delta\mathcal{H}_0=\int_{\Delta t}\omega(I)\dot{I}dt=\epsilon\sin\nu t$$

$$=\frac{\epsilon}{\nu}\int_{\Delta t}\left(\frac{d}{dt'}\delta_{2\pi}(t'-t)\right)\cos\nu t'dt', \tag{9}$$

where $\delta_{2\pi}(z)$ is the 2π periodic δ function. Comparing integrands from the left- and right-hand sides of Eq. (9), we obtain for the arbitrary time scale Δt the following equations of motion for the action-angle pair:

$$\dot{I}=\frac{\epsilon}{\nu}\omega(I)\,\delta'_{2\pi}(\theta)\cos\nu t,$$

$$\dot{\theta}=\omega(I), \tag{10}$$

where $\delta'_{2\pi}(\theta)\equiv(d/d\theta)\delta_{2\pi}(\theta)$. These equations of motion are not Hamiltonian. To obtain such equations, the perturbation of the form $(\epsilon/\nu)\omega'(I)\delta_{2\pi}(\theta)\cos\nu t$ is added to the second equation of Eq. (10). Finally, chaotic dynamics is described by the following effective Hamiltonian:

$$\mathcal{H}=\mathcal{H}_0(I)-\frac{\epsilon}{\nu}\omega(I)\,\delta_{2\pi}(\theta)\cos\nu t. \tag{11}$$

It is simple to show from Eqs. (10) and (11) that an influence of adding the perturbative term is negligibly small, when $\epsilon\omega'(I)/\omega(I)\ll 1$. It should be stressed that the effective Hamiltonian (11) describes the same resonances taking place in Eq. (5) with the same threshold of chaos. It follows from Eqs. (5) and (11) that $\nu T(\mathcal{E}_r)=2\pi l$ and $l\,\dot{\theta}(I_r)=l\,\omega(I_r)=\nu$, where I_r is the resonant action corresponding to the lth resonant and $\mathcal{E}_r=\mathcal{E}(I_r)$ is corresponding resonant energy determined from Eq. (8). The following expansion $\delta_{2\pi}(\theta)=2\Sigma_{l=0}^{\infty}\cos\theta-1$ is used as well. This Hamiltonian can be obtained for an arbitrary dispersion law with the period $T_c(\mathcal{E})$, because in the procedure (10) and (11) an explicit form of $\mathcal{H}_0$ and, consequently, of $T_c(\mathcal{E})$ was not used [17].

The Hamiltonian formulation of the problem allows semiclassical quantization in the action-angle formulation: $I\to\hat{I}=\tilde{h}\hat{n}=-i\tilde{h}\partial/\partial\theta$. Here $\tilde{h}$ is a dimensionless Planck constant defined from the number of quanta in the magnetic field flux. The semiclassical approximation requires that the width of the perturbative potential is larger than the de Broglie wavelength [18]. In this connection it is necessary to restrict summation in the Fourier expansion of the $\delta_{2\pi}(\theta)$ potential. We have understood the δ function as an approximation of a potential with width of a spike that equals to $2\pi/N$: $\delta_N(\theta)=2\Sigma_{k=0}^{N}\cos k\theta-1$, where $\delta_N(\theta)$ tends to $\delta_{2\pi}(\theta)$ as N tends to infinity and $1/\tilde{h}\gg N\gg 1$.

The Hamiltonian (11) can be rewritten in the Hermitian form

$$\hat{\mathcal{H}}=\hat{\mathcal{H}}_0(\hat{n})-\frac{\epsilon}{\nu}[\omega(\hat{n})\,\delta_N(\theta)+\text{H.c.}]\cos\nu t, \tag{12}$$

where H.c. is the Hermitian conjugation and $\hat{\mathcal{H}}_0(\hat{n})$ is the quantized unperturbed Hamiltonian in Eq. (11).

III. THE VELOCITY OPERATOR

The velocity operator $\hat{v}(0)$ can be determined from $\mathcal{H}_0$ by its standard definition: $v(k)=\partial\varepsilon(k)/\partial(\hbar k)$, where k is a wave number while $\varepsilon(k)$ is an energy dispersion law. It is reasonable to believe that it coincides *locally* with $\mathcal{H}_0$ with corresponding redefinition of the parameters due to the presence of the magnetic field $\vec{H}=(0,0,H)$. Therefore after Peierls substitution [19] we can write in the Landau gauge for two-dimensional motion that $\varepsilon(k_xa,k_yb,H_z)=\varepsilon(a/\hbar p_x;\ (b/\hbar)(p_y-[eH/c]x))$, where a,b are the periods of lattice in the x and y directions, respectively. The dispersion law ε can be rewritten approximately in the following form:

$$\varepsilon(k,H_z)\approx\varepsilon_0+\frac{\varepsilon_2}{2}\left(\frac{a}{\hbar}\right)^2p_x^2+\frac{\varepsilon_2}{2}\left(\frac{b}{\hbar}\right)^2\times\left(p_y-\frac{eH}{c}x\right)^2-\frac{\varepsilon_4}{4}\left[\left(\frac{a}{\hbar}\right)^2\frac{p_x^2}{2}+\frac{1}{2}\left(\frac{b}{\hbar}\right)^2\times\left(p_y-\frac{eH}{c}x\right)^2\right]^2, \tag{13}$$

where $\varepsilon_0,\varepsilon_2,\varepsilon_4$ are parameters of the expansion. It is convenient to introduce dimensionless variables in the following form:

$$p=\frac{p_xa}{\hbar},\quad q=\frac{b}{\hbar}\left(\frac{eH}{c}x-p_y\right)=\frac{eHb}{c\hbar}(x-x_0),$$
$$x_0=\frac{b}{\hbar}p_y,\quad \omega^*t\to t,\quad \frac{\nu}{\omega^*}\to\nu,\quad \omega^*=\frac{\varepsilon_2abeH}{\hbar^2c}. \tag{14}$$

Here the expression for ω^* is just the definition of the cyclotron frequency for Eq. (3). The dimensionless Plank constant is also determined from Eq. (14) by the Poisson brackets for p and q :

$$\tilde{h}=\hbar\{q,p\}_P=\hbar\frac{\partial q}{\partial x}\frac{\partial p}{\partial p_x}=\frac{Hab}{\hbar c/e}\equiv 2\pi\frac{\Phi}{\Phi_0}, \tag{15}$$

where Φ is the magnetic field flux through the cell ab, and $\Phi_0=2\pi\hbar c/e$. Introducing new variables

$$p=i\sqrt{\frac{\tilde{h}}{2}}(k-k^{\dagger}),\quad q=\sqrt{\frac{\tilde{h}}{2}}(k+k^{\dagger}),\quad I=\tilde{h}k^{\dagger}k, \tag{16}$$

and defining the Hamiltonian as $\varepsilon-\varepsilon_0/\hbar\omega^*\to\mathcal{H}_0$ with $\Omega=\varepsilon_2/\hbar\omega^*$, $\mu=\varepsilon_4/\hbar\omega^*$, one obtains the relation between the dispersion law in the local form (13) and the Hamiltonian $\mathcal{H}_0$. Therefore the velocity is $v\equiv v^+ + v^-=\dot{q}=\partial\mathcal{H}_0/\partial p$, where

$$v^+=i\sqrt{2\tilde{h}}\omega(I)k,$$
$$v^-=-i\sqrt{2\tilde{h}}k^{\dagger}\omega(I). \tag{17}$$

For the semiclassical quantization, we can consider $k^{\dagger},k$ as a creation and an annihilation operators that take form in the $e^{in\theta}=\langle\theta|n\rangle$ representation

$$\hat{k}^{\dagger}=\sqrt{\hat{n}}e^{i\theta},\quad \hat{k}=e^{-i\theta}\sqrt{\hat{n}},\quad \hat{k}^{\dagger}\hat{k}=\hat{n}=-i\frac{\partial}{\partial\theta},$$

$$\hat{k}^{\dagger}|n\rangle=\sqrt{\hat{n}}e^{i\theta}e^{in\theta}=\sqrt{n+1}|n+1\rangle, \tag{18}$$

$$\hat{k}|n\rangle=\sqrt{n}|n-1\rangle,\quad \hat{k}^{\dagger}\hat{k}|n\rangle=n|n\rangle.$$

One obtains also from Eq. (18) that

$$\langle m'|\hat{v}^{-}|m\rangle=-i\sqrt{2\tilde{h}}\omega(m)\sqrt{m+1}\,\delta_{m',m+1},$$
$$\langle m|\hat{v}^{+}|m'\rangle=i\sqrt{2\tilde{h}}\omega(m'-1)\sqrt{m'}\,\delta_{m,m'-1}, \tag{19}$$

where $\delta_{i,j}$ is the Kronecker δ symbol.

IV. DIAGONALIZATION OF THE FLOQUET OPERATOR

The Hamiltonian (12) is periodic in time, hence the Floquet theory can be used for the analysis. The Shrödinger equation

$$i\tilde{h}\frac{\partial}{\partial t}\psi(\theta,t)=\hat{\mathcal{H}}(\theta,t)\psi(\theta,t) \tag{20}$$

is rewritten for the Floquet operator

$$\hat{F}=-i\tilde{h}\frac{\partial}{\partial t}+\hat{\mathcal{H}}(\hat{n},\theta,t)=-i\tilde{h}\frac{\partial}{\partial t}+\hat{\mathcal{H}}_0(\hat{n})+\hat{V}(\hat{n},\theta,t) \tag{21}$$

in the form of the eigenvalue problem

$$\hat{F}u_{\alpha}(\theta,t)=\tilde{h}\lambda_{\alpha}u_{\alpha}(\theta,t), \tag{22}$$

due to the Floquet theorem

$$\psi(\theta,t)=e^{-i\lambda_{\alpha}t}u_{\alpha}(\theta,t), \tag{23}$$

where $|u_{\alpha}(t)\rangle\equiv u_{\alpha}(\theta,t)$ are periodic in time with the period $2\pi/\nu$ quasienergy eigenfunctions, and λ_{α} are corresponding eigenvalues. Therefore, $\mathcal{R}$ in Eqs. (1) and (2) can be determined as follows:

$$\begin{aligned}\mathcal{R}&=\sum_{n}\langle n|\hat{\rho}\hat{v}(t)\hat{v}(0)|n\rangle+\text{c.c.}\\&=\sum_{\alpha,\beta}\sum_{n,n_1}\rho_{n,n_1}\langle n_1|u_{\alpha}(0)\rangle\langle u_{\alpha}(0)|\hat{U}^{\dagger}(t)\hat{v}(0)\hat{U}(t)\\&\quad\times|u_{\beta}(0)\rangle\langle u_{\beta}(0)|\hat{v}(0)|n\rangle+\text{c.c.}\\&=\sum_{\alpha,\beta}e^{-i(\lambda_{\beta}-\lambda_{\alpha})t}\sum_{n,n_1}\rho_{n,n_1}\langle n_1|u_{\alpha}(0)\rangle\langle u_{\alpha}(t)|\hat{v}|u_{\beta}(t)\rangle\\&\quad\times\langle u_{\beta}(0)|\hat{v}|n\rangle+\text{c.c.},\end{aligned} \tag{24}$$

where c.c. means complex conjugation and $\rho_{n,n_1}=\langle n|\hat{\rho}|n_1\rangle$. It is used here that for the moment $t=0$, $\psi(\theta,0)=u_{\alpha(\beta)}(\theta,0)\equiv u_{\alpha(\beta)}(0)$ and $\hat{U}(t)\psi(\theta,0)=e^{-i\lambda_{\alpha(\beta)}t}u_{\alpha(\beta)}(\theta,t)$, where $\alpha(\beta)$ means here and in the following either α or β. To calculate the matrix elements in Eq. (24), one needs to find $\lambda_{\alpha(\beta)}$ and $u_{\alpha(\beta)}$. To that end the eigenvalue equation for the Floquet operator can be projected onto a basis of ''photon states'' [20,16]. The reason for the occurrence of the ''photon states'' or quasiresonances [21] is that the unperturbed spectrum E_n is a slowly varying function of n [see Eq. (8)]. There are many transition frequencies, and many near-resonances (quasiresonances) take place due to the external field $\hat{V}(\hat{n},\theta,t)$. Thus the eigenvalue $E(n)$ of the unperturbed Hamiltonian $\mathcal{H}_0$ can be expanded around a large value n_0 to the first order $E(n)\approx E(n_0)+\tilde{h}\omega(n_0)\Delta n$, where $\Delta n=n-n_0$. Then the eigenvalue equation (22) for this approximation corresponds to the eigenvalue problem for a ''linear kicked rotor'' [16,22], which is an exactly solvable model [23] with quasienergies $\bar{\lambda}=l\omega(\Delta n)(\text{mod}\nu)$. The index $\alpha(\beta)$ is omitted in what follows. The corresponding eigenstates in the n representation can be expressed as a composition of a chain of peaks separated by the energy of a photon of the external field $\tilde{h}\nu$. These peaks correspond to the quasiresonances and are described by a sinc function of the form

$$Q_j^{\lambda}(n)=\frac{\sin[\pi(n-n_j+\delta_j)]}{\pi(n-n_j+\delta_j)], \tag{25}$$

where n_j corresponds to the center of the jth peak and δ_j characterizes its precise shape. These are determined from the linearized model by the following expressions: $E_{n_j}=E(n_0)+\tilde{h}\omega(n_0)\Delta n_j\approx\tilde{h}\bar{\lambda}+\tilde{h}\nu j-\tilde{h}\omega(n_0)\delta_j$ with $\tilde{h}\bar{\lambda}=E(n_0)-\tilde{h}\omega(n_0)n_0$ and

$$n_j=\text{int}\left\{\frac{I(j)}{\tilde{h}}\right\},\quad \delta_j=\text{frac}\left\{\frac{I(j)}{\tilde{h}}\right\}, \tag{26}$$

where int$\{x\}$ is the integer part of x and frac$\{x\}$ is its fractional part [16]. We can obtain a solution for the origin nonlinear problem (22) by matching quasiresonances (25) for different energies [16,22]. Thus the quasiresonances determine the energy E_{n_j} on the ladder of states n_j analogous to the linear counterpart by the form

$$E_{n_j}=\tilde{h}\lambda+\tilde{h}\nu j-\tilde{h}\delta_j\omega(n_j). \tag{27}$$

The eigenvalue λ corresponds to Eq. (22) and sets the origin of the ladder for the corresponding quasienergy eigenfunction, and the quasiresonance function (25) are numbered by their position j on the ladder, where n_j is the peak of the position and the detuning $|\delta_j|<1/2$ satisfies Eq. (27). The quasienergy eigenstate can be approximated by $\langle n|u_{\lambda}\rangle=\Sigma_j A_j Q_j(n)$, where A_j stands to be determined. To this end, the following procedure is carried out. Eigenfunctions for the unperturbed Floquet operator $\hat{F}_0=-i\tilde{h}\partial/\partial t+\mathcal{H}_0$ are

$$|n,j\rangle=e^{in\theta}e^{-i\nu jt}\equiv|n\rangle|j\rangle \tag{28}$$

with corresponding eigenvalues $E_n-\tilde{h}\nu j$. Using the definition $\phi_{n,j}^{\lambda_{\alpha(\beta)}}=\langle n,j|u_{\lambda_{\alpha(\beta)}}(\theta,t)\rangle$ and omitting for shortness the

index $\alpha(\beta)$, we project the eigenvalue equation for $\hat{F}$ on the basis $\langle j,n|$ and obtain the eigenvalue equation for the coefficients $\phi^{\lambda}_{n,j}$ in the form

$$(E_n-\tilde{h}\nu j)\phi^{\lambda}_{n,j}+\frac{\epsilon}{2\nu}\sum_{n'}\langle n|\omega(\hat{n})\delta_{2\pi}(\theta)+\delta_{2\pi}(\theta)\omega(\hat{n})|n'\rangle \times[\phi^{\lambda}_{n',j+1}+\phi^{\lambda}_{n',j-1}]=\tilde{h}\lambda\phi^{\lambda}_{n,j}. \quad (29)$$

In the unperturbed case, $\epsilon=0$ and $\tilde{h}\lambda=E_{n_j}-\tilde{h}\nu j$, with eigenstates being δ functions at sites n_j. The perturbation mixes different values of j with approximately the same λ to form a ladder of quasiresonances $Q^{\lambda}_j(n)$ (25). The coefficients are constructed as the following combination of the $Q^{\lambda}_j(n)$: $\phi^{\lambda}_{n,j}=A^{\lambda}_j Q^{\lambda}_j(n)$. Using this fact and the explicit expressions (25) and (27), the eigenvalue equation takes the form

$$-\tilde{h}\delta_j\omega(n_j)Q^{\lambda}_j(n_j)A^{\lambda}_j+\frac{\epsilon}{4\pi\nu}\sum_{n'}[\omega(n_j)+\omega(n')] \times[Q^{\lambda}_{j+1}(n')A^{\lambda}_{j+1}+Q^{\lambda}_{j-1}(n')A^{\lambda}_{j-1}]\approx 0. \quad (30)$$

To sum up over n' in Eq. (30), we represent the frequency $\omega(n')$ in the form of a Cauchy integral:

$$\omega(l)=\frac{1}{2\pi i}\int_{\mathcal{G}}\frac{d\xi\omega(\xi)}{\xi-l}, \quad (31)$$

where $\mathcal{G}$ is a contour of integration along the real axis. Then we obtain from Eq. (25)

$$\sum_{n'}\frac{1}{\xi-n'}Q^{\lambda}_k(n')=\sum_{l=-\infty}^{\infty}\frac{(-1)^l\sin\pi\delta_k}{\pi(l+\delta_k)(\xi-l-n_k)}=\frac{1}{\xi+\delta_k-n_k} +\frac{\sin\pi\delta_k}{\sin[\pi(\xi-n_k)](\xi-n_k+\delta_k)}, \quad (32)$$

where the change of variables $l=n'-n_k$ is used. After integration (31) one obtains the following expression: $\omega(n_{j\pm1})\sin\pi\delta_{j\pm1}/\pi\delta_{j\pm1}$. It is simple to see that $\omega(n_{j\pm1})\approx\omega(n_j)$ when $\tilde{h}\nu\omega'/\omega^2\ll 1$, which is consistent also with the inequality $\epsilon\omega'/\omega\ll 1$ used for Eq. (11) and the condition for the semiclassical approximation for the perturbation $\tilde{h}\nu\ll\epsilon$. Redefining $C_j=(1+\sin\pi\delta_j/\pi\delta_j)A^{\lambda}_j$, and taking into account that $\Sigma_{n'}Q_j(n')=1$, one obtains finally from Eq. (30) the Anderson-like equation

$$C_{j+1}+C_{j-1}+\frac{1}{\kappa}\mathcal{U}(\delta_j)C_j=\Lambda C_j \quad (33)$$

with eigenvalue $\Lambda\to 0$, and $\kappa=\epsilon/4\tilde{h}\nu\pi$, while $\mathcal{U}(\delta_j)=\delta_j\sin\pi\delta_j(\delta_j+\sin\pi\delta_j)^{-1}$ is a pseudorandom potential [16]. For large j we conclude from Eq. (27) that $E_{n_j}=\tilde{h}\nu j$. It follows from Eqs. (8) and (27) that $\delta_j=\mathrm{frac}\{\zeta\tilde{h}\nu^2j^2/2\}=\mathrm{frac}\{(\mathcal{K}/2\kappa)j^2\}$. These numbers form a pseudorandom sequence as a function of quasiresonance number j [24] with a uniform distribution in $[-\frac{1}{2},\frac{1}{2}]$. Hence, the solution corresponds to the exponential localization of A^{λ}_j with some mean localization length $\langle\gamma\rangle^{-1}\sim 16\kappa^2$ as follows: $A^{\dagger\lambda}_j A^{\lambda}_l\sim(1+\sin\pi\delta_j/\pi\delta_j)^{-1}(1+\sin\pi\delta_l/\pi\delta_l)^{-1}e^{-\langle\gamma\rangle|l-j|}$ [25].

V. THE KINETIC COEFFICIENT

Solution of the eigenvalue problem for the Floquet operator enables us to express the correlation functions $\mathcal{R}$ in explicit form. The wave function takes the form $|u_{\alpha(\beta)}(\theta,t)\rangle=\Sigma_{m,j}Q^{\lambda_{\alpha(\beta)}}_j(m)A^{\lambda_{\alpha(\beta)}}_j e^{im\theta}e^{-i\nu jt}$. Taking into account the explicit expressions for the wave functions and Eq. (19) we obtain from Eq. (24) the following expression for $\mathcal{R}$:

$$\mathcal{R}=\sum_{\alpha,\beta}\sum_{n}\sum_{n_1}\sum_{m}\sum_{j,k,l}\sum_{r}e^{-i(\lambda_\beta-\lambda_\alpha)}A^{\dagger\lambda_\alpha}_j \times A^{\lambda_\alpha}_k A^{\dagger\lambda_\beta}_l A^{\lambda_\beta}_j d_{r,1}Q^{\lambda_\beta}_l(n-d_{r,1})Q^{\lambda_\alpha}_k(n_1) \times\rho_{n,n_1}Q^{\lambda_\alpha}_j(m)\omega(m+d_{r,2})\omega(n-1-d_{r,3}) \times\sqrt{m+d_{r,4}}\sqrt{n+1-d_{r,5}}Q^{\lambda_\beta}_j(m+d_{r,6})+\text{c.c.} \quad (34)$$

The 4×6 matrix $d_{r,s}$ is introduced for shortness of notation and it is shown in Appendix A. Using Eq. (25) and the auxiliary expressions of Eqs. (31) and (32), one can carry out the sums over n,n_1, and m (see Appendix A). Summing over m, we obtain from Eqs. (25) and (27):

$$\sum_{m}\omega(m+d_{r,2})\sqrt{m+d_{r,4}}Q^{\lambda_\alpha}_j(m)Q^{\lambda_\beta}_j(m+d_{r,6}) =\frac{\sin\pi\delta_j}{\pi\delta_j}\omega[n_j(\alpha)-\delta_j(\alpha)+d_{r,2}]\sqrt{n_j(\alpha)-\delta_j(\alpha)+d_{r,4}} \times\frac{\sin\pi[n_j(\alpha)-\delta_j(\alpha)-n_j(\beta)+\delta_j(\beta)+d_{r,6}]}{\pi[n_j(\alpha)-\delta_j(\alpha)-n_j(\beta)+\delta_j(\beta)+d_{r,6}]}, \quad (35)$$

where

$$n_j[\alpha(\beta)]=\frac{\lambda_{\alpha(\beta)}}{\omega}+j\frac{\nu}{\omega}-\delta_j[\alpha(\beta)]. \quad (36)$$

The maximal contribution of Eq. (35) to sums over α and β in Eq. (34) is when the denominator in Eq. (35) vanishes: $n_j(\alpha_c)-\delta_j(\alpha_c)-n_j(\beta_c)+\delta_j(\beta_c)\pm 1=0$. Then we obtain from Eq. (36) an expression for the quasienergy difference:

$$\lambda_{\alpha_c}-\lambda_{\beta_c}=2[\delta_j(\alpha_c)-\delta_j(\beta_c)]\omega\mp\omega\equiv\Delta\lambda_{j,c}\omega\mp\omega. \quad (37)$$

Finally, the expression for the correlation function $\mathcal{R}$ (see Appendix A) reads

$$\mathcal{R}\approx-2\sum_{c}\sum_{j,k,l}\{G^{+}_c(j,k,l)\cos(\omega t-\Delta_{j,c}\omega t) +G^{-}_c(j,k,l)\cos(\omega t+\Delta_{j,c}\omega t)\}\exp\{-\langle\gamma\rangle(|j-k| +|l-j|)\}, \quad (38)$$

where $G^{\pm}_c(j,k,l)$ are slowly varying functions of j,k,l, and their explicit form is presented in Appendix A.

The expression (38) depends strongly on the localization length $1/\langle\gamma\rangle$. When $\langle\gamma\rangle\to 0$ exponential truncation of the sum over the index j does not take place since then $e^{-\langle\gamma\rangle j}\to 1$. In this case the sum over j can be estimated due to the oscillating terms. Taking into account that $G_c^{\pm}(j,k,l)$ are slowly varying functions, and rearranging $\Delta_{j,c}$ as an increasing set ranging from -2 to 2, we obtain approximately that $\Sigma_j \sin\Delta_{j,c}\omega t\to 0$ and $\Sigma_j \cos\Delta_{j,c}\omega t\propto(1/\omega)\,\delta(t)$. Then we obtain from Eq. (1) that $K(z)=K$, where K is the constant value of the kinetic coefficient. The oscillating behavior of K as a function of the magnetic field disappears due to classical chaotic dynamics that is realized in the fact that the localization length goes to infinity.

In the opposite case when localization is strong enough to cut exponentially the sum over j,k,l, then only a few terms of the row should be taken into account. In this case we obtain (see Appendix B) that the main term is proportional to $e^{-\langle\gamma\rangle}$ when $j=l\pm 1$ and $k=j$, or $j=k\pm 1$ and $l=j$. This term, which contributes resonantly to the kinetic coefficient $K(z)$, obtained in Appendix B, reads

$$2e^{-\langle\gamma\rangle}\sum_c\ [\mathcal{L}_c^{+}\cos(\omega t-\Delta_{1,c}\omega t)+\mathcal{L}_c^{-}\cos(\omega t+\Delta_{1,c}\omega t)]. \tag{39}$$

Hence, we obtain the kinetic coefficient from Eq. (1) in the form

$$K(z)=\sum_c\left[\frac{\tau e^{-\langle\gamma\rangle}\mathcal{L}_c^{+}}{1+\tau^2(z+\omega-\Delta_{1,c}\omega)^2}+\frac{\tau e^{-\langle\gamma\rangle}\mathcal{L}_c^{+}}{1+\tau^2(z-\omega+\Delta_{1,c}\omega)^2}+\frac{\tau e^{-\langle\gamma\rangle}\mathcal{L}_c^{-}}{1+\tau^2(z+\omega+\Delta_{1,c}\omega)^2}+\frac{\tau e^{-\langle\gamma\rangle}\mathcal{L}_c^{-}}{1+\tau^2(z-\omega-\Delta_{1,c}\omega)^2}\right] \tag{40}$$

The oscillating behavior of the coefficient $K(z)$ as a function of the magnetic field reflects in the poles of Eq. (40). When the magnetic field is monotonically tuned, the quasienergy differences $\pm(\lambda_{\alpha(c)}-\lambda_{\beta(c)})$ coincide with z repeatedly. For example, for $z=0$ Eq. (40) corresponds to dc conductance and the resonant condition fulfills when $\lambda_{\alpha(c)}=\lambda_{\beta(c)}$ for some value of H. As it follows from Eqs. (27) and (36) when H changes, the spectrum $E(n)$ changes and the magnitude of the pseudorandom numbers $\{\delta_j\}$ changes as well. Therefore, when the magnetic field is tuned the denominators in Eq. (40) change violating the resonance conditions and corresponding to them repeatedly. The expression (40) describes the response of the quantum system that is chaotic in the classical limit. It reflects an essential difference between classical and quantum dynamics, where dynamical localization of classical chaos plays a crucial role. It is also necessary to admit that this result is obtained in the framework of the nonperturbative approach, where the strength of the high-frequency field ϵ is not a perturbation parameter but the value of $e^{-\langle\gamma\rangle}\propto e^{-1/\epsilon^2}$ is the small parameter for the expansion carried out to obtain Eq. (40).

VI. SUMMARY

We must justify the use of the Kubo formula (1) for the response of the chaotic system to an infinitesimal perturbation. The Kubo formula assumes that $\hat{\rho}$ is the time-independent operator and it corresponds to the equilibrium. But the chaotic system is not in equilibrium. Indeed, electrons interacting resonantly with the alternating field in the skin layer acquire energy from the field due to chaotic dynamics and then transfer it to the total electron system due to the electron-electron collisions. In this sense these collisions determine the mean free time τ. For an isolated system this process increases the temperature of the sample. To keep the temperature constant, a heat flow out of the system is required. The temperature change due to this chaotic dynamics is very small, because a number of electrons taking part in the cyclotron resonance dynamics $n_{c.r.}$ is much smaller than the total number of conducting electrons n: $n_{c.r.}/n\approx\Delta/l\approx 10^{-4}$, where Δ is the skin layer depth, while l is a linear size of the sample. Therefore the transferred energy per a conducting electron is negligibly small and consequently the temperature change is negligible as well ($\Delta T<1$ K). In this sense the total electron system can be considered as a thermal bath for the chaotic dynamics system. Hence, we can assume that $\hat{\rho}$ describes a system near the equilibrium with the constant temperature, and it is a time-independent operator. These speculations may justify the use of Eq. (1) for the kinetic coefficient, as well as the Fourier transform procedure carried out in Eq. (40).

This analysis can be considered as a sum paradigm of a quantum system with $\tilde{h}\ll 1$ and in this case the relevant description of the kinetic features of the system is the classical one. When the classical counterpart of the system is chaotic, the quantum nature of the system is reflected in kinetic features, as it is shown here. The nature of this quantum kinetics is totally different from the classical one. This difference is a result of quantum saturation of classical chaotic diffusion in the energy space.

This phenomenon can be observed experimentally in metals or semimetals under anomalous skin effect at helium temperature. For example, in bismuth under anomalous skin effect conditions at Helium temperatures (see, for example, Ref. [26]). In this case $H\sim 10^3$ Oe, $\nu\sim 10^{11}$ sec^{-1}, and $\epsilon=\Delta E/\hbar\omega^*$, while $\zeta=\Delta T/\Delta\mathcal{E}\approx 2\pi\hbar\omega^*/\nu\Delta E$. Hence it follows that criterion for chaos $\mathcal{K}=\nu\epsilon\zeta\approx 2\pi>1$ is fulfilled. The condition $\omega_c\tau\gg 1$ still holds.

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APPENDIX A

To obtain Eq. (38) from Eq. (34) we need to sum over n_1,n,m,α, and β. Using Eqs. (31) and (32), one obtains that

$$\sum_{n_1} \rho_{n,n_1} Q_k^{\lambda\beta}(n_1) \equiv \sum_{n_1} \rho(n,n_1) Q_k^{\lambda\beta}(n_1) = \frac{\sin \pi\delta_k(\beta)}{\pi\delta_k(\beta)} \rho(n, n_k(\beta) - \delta_k(\beta)). \tag{A1}$$

Analogously, sums over m and n read

$$\sum_m Q_j^{\lambda\alpha}(m)\omega(m+d_{r,2})\sqrt{m+d_{r,4}} Q_j^{\lambda\beta}(m+d_{r,6}) = \frac{\sin \pi\delta_j(\alpha)}{\pi\delta_j(\alpha)} \omega[n_j(\alpha) - \delta_j(\alpha) + d_{r,2}] \sqrt{n_j(\alpha) - \delta_j(\alpha) + d_{r,4}} Q_j^{\lambda\beta}[n_j(\alpha) - \delta_j(\alpha) + d_{r,6}], \tag{A2}$$

where

$$Q_j^{\lambda\beta}[n_j(\alpha) - \delta_j(\alpha) + d_{r,6}] = \frac{\sin \pi[n_j(\alpha) - \delta_j(\alpha) - n_j(\beta) + \delta_j(\beta) + d_{r,6}]}{\pi[n_j(\alpha) - \delta_j(\alpha) - n_j(\beta) + \delta_j(\beta) + d_{r,6}]} \tag{A3}$$

and

$$\sum_n Q_l^{\lambda\beta}(n - d_{r,1})\omega(n - 1 - d_{r,3})\sqrt{n + 1 - d_{r,5}} \rho(n, n_k(\beta) - \delta_k(\beta)) = \frac{\sin \pi\delta_l(\beta)}{\pi\delta_l(\beta)} \omega[n_l(\beta) - \delta_l(\beta) - 1 - d_{r,3} + d_{r,1}] \sqrt{n_l(\beta) - \delta_l(\beta) + 1 - d_{r,5} + d_{r,1}} \rho(n_l(\beta) - \delta_l(\beta) + d_{r,1}, n_k(\beta) - \delta_k(\beta)). \tag{A4}$$

Here $d_{r,s}$ is 4×6 matrix introducing for shortness of the notation. It reads

$$d = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & -1 & -1 & 0 & 0 & -1 \end{pmatrix}, \tag{A5}$$

where $d_{r,1} - 1 - d_{r,3} = d_{r,3}$ and $d_{r,1} + 1 - d_{r,5} = d_{r,5}$. As it is seen from Eqs. (35), (A3) and (A5), the main contribution in the summation over α and β is due to the denominators:

$$n_j(\alpha_c) - \delta_j(\alpha_c) - n_j(\beta_c) + \delta_j(\beta_c) \pm 1 = 0, \tag{A6}$$

such that $Q_j^{\lambda\beta_c}[n_j(\alpha_c) - \delta_j(\alpha_c) + d_{r,6}] = 1$. Let us denote by

$$-G_c^+(j,k,l) = \sum_{r=1}^{2} d_{r,1} \rho(n_l(\beta_c) - \delta_l(\beta_c) + d_{r,1}, n_k(\beta_c) - \delta_k(\beta_c)) \omega[n_j(\alpha_c) - \delta_j(\alpha_c) + d_{r,2}] \sqrt{n_j(\alpha_c) - \delta_j(\alpha_c) + d_{r,4}} \omega[n_l(\beta_c) - \delta_l(\beta_c) + d_{r,3}] \sqrt{n_l(\beta_c) - \delta_l(\beta_c) + d_{r,5}} \frac{\sin \pi\delta_j(\alpha_c)}{\pi\delta_j(\alpha_c)} \times \left(1 + \frac{\sin \pi\delta_j(\alpha_c)}{\pi\delta_j(\alpha_c)}\right)^2 \frac{\sin \pi\delta_k(\beta_c)}{\pi\delta_k(\beta_c)} \left(1 + \frac{\sin \pi\delta_k(\beta_c)}{\pi\delta_k(\beta_c)}\right) \frac{\sin \pi\delta_l(\beta_c)}{\pi\delta_l(\beta_c)} \left(1 + \frac{\sin \pi\delta_l(\beta_c)}{\pi\delta_l(\beta_c)}\right), \tag{A7}$$

and for the sum over r from 3 to 4 by $-G_c^-(j,k,l)$. Then one obtains Eq. (38) from Eqs. (34), (37) and (A7).

APPENDIX B

As it has been mentioned, $\mathcal{R}$ depends strongly on the localization length $\langle\gamma\rangle^{-1}$. In the case of $\langle\gamma\rangle \to 0$ one needs to evaluate the following sums:

$$\Sigma = \sum_j G_c^{\pm}(j,k,l) \begin{Bmatrix} \sin \Delta_{j,c}\omega t \\ \cos \Delta_{j,c}\omega t \end{Bmatrix}, \tag{B1}$$

where $G_c^{\pm}(j,k,l) \propto \sqrt{n_j} \propto \sqrt{j}$ are slowly varying functions of j. Rearranging these sums such that the pseudorandom values of $\Delta_{j,c}$ range in order from -2 to 2, we obtain that the new set of $\{j\}$ and hence $G_c^{\pm}(j,k,l)$ corresponded to this ordering become pseudorandom as well. Then we obtain, approximately, that

$$\Sigma \propto \langle G^{\pm}\rangle \sum_j \begin{Bmatrix} \sin \Delta_{j,c}\omega t \\ \cos \Delta_{j,c}\omega t \end{Bmatrix} \to \langle G^{\pm}\rangle \begin{Bmatrix} 0 \\ \frac{1}{\omega}\delta(t) \end{Bmatrix}, \tag{B2}$$

where $\langle G^{\pm}\rangle = \lim_{N\to\infty}(1/N)\Sigma_j^N G_c^{\pm}(j,k,l)$.

In the opposite case when $e^{-\langle\gamma\rangle} \ll 1$ one can take into account only the first two terms for sums over j,k,l for $\exp\{-\langle\gamma\rangle[(j-k)+(l-j)]\}$:

$$\sum_{j,k,l} \{\cdots\} \exp\{-\langle\gamma\rangle[(j-k)+(l-j)]\} \approx 1 \times \sum_{j,k,l} \{\cdots\} + e^{-\langle\gamma\rangle} \sum_{j,k,l} \{\cdots\}. \tag{B3}$$

This imposes the following condition on j,k,l in the sums: (1) $j=k=l$ for the first term and (2) $j=k\pm 1, j=l$, or $j=l$

$\pm 1, j=k$ for the second term. Then we obtain from Eqs. (38), (B1), and (B2) that the first term does not contribute resonantly:

$$\sum_j G_c^{\pm}(j,j,j)\begin{Bmatrix}\sin\Delta_{j,c}\omega t\\ \cos\Delta_{j,c}\omega t\end{Bmatrix}\to\langle G^{\pm}\rangle\begin{Bmatrix}0\\ \dfrac{1}{\omega}\delta(t)\end{Bmatrix}. \quad \text{(B4)}$$

For the condition (2) we obtain

$$\begin{aligned}\tilde{\mathcal{R}}=e^{-\langle\gamma\rangle}\sum_{j=2}\sum_{s=j-1}&\{[G_c^+(j,j,s)+G_c^+(j,s,j)]\\ &\times\cos(\omega t-\Delta_{j,c}\omega t)+[G_c^-(j,j,s)+G_c^-(j,s,j)]\\ &\times\cos(\omega t+\Delta_{j,c}\omega t)\}+e^{-\langle\gamma\rangle}\sum_{j=1}\sum_{s=j+1}\{[G_c^+(j,j,s)\\ &+G_c^+(j,s,j)]\cos(\omega t-\Delta_{j,c}\omega t)+[G_c^-(j,j,s)\\ &+G_c^-(j,s,j)]\cos(\omega t+\Delta_{j,c}\omega t)\}. \quad \text{(B5)}\end{aligned}$$

To complete summation over j in the first sum, we add and subtract the following terms:

$$e^{-\langle\gamma\rangle}\mathcal{L}_c^{\pm}\cos(\omega t\pm\Delta_{1,c}\omega t), \quad \text{(B6)}$$

where we use the notation

$$\mathcal{L}_c^{\pm}=\sum_{s=1}[G_c^{\pm}(1,1,s)+G_c^{\pm}(1,s,1)]. \quad \text{(B7)}$$

Then we obtain from Eqs. (B5), (B7)

$$\begin{aligned}\tilde{\mathcal{R}}=e^{-\langle\gamma\rangle}\sum_{j=1}\sum_{s=j\pm1}{}'&\{[G_c^+(j,j,s)+G_c^+(j,s,j)]\cos(\omega t\\ &-\Delta_{j,c}\omega t)+[G_c^-(j,j,s)+G_c^-(j,s,j)]\cos(\omega t+\Delta_{j,c}\omega t)\}\\ &-e^{-\langle\gamma\rangle}\mathcal{L}_c^{+}\cos(\omega t-\Delta_{1,c}\omega t)-e^{-\langle\gamma\rangle}\mathcal{L}_c^{-}\cos(\omega t\\ &+\Delta_{1,c}\omega t). \quad \text{(B8)}\end{aligned}$$

The prime in the sum over s means that the terms with $s=0$ are omitted. Again, summing over j one obtains nonresonant terms as in Eqs. (B1) and (B2). The last two terms in Eq. (B8) with $j=1$ contribute resonantly, yielding the expressions (39) and (40).

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